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USING STATE SPACE MODELS AND COMPOSITE ESTIMATION

to measure the effects of telephone interviewing on labour force estimates

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Abstract

This paper describes the use of composite estimation and state space modelling techniques for analysis of data from a repeating survey. The techniques take account of common sample between successive months and the resulting autocorrelation structure of the sampling error. The techniques are illustrated by an investigation of the effect of introducing telephone interviewing in the Australian Labour Force Survey.

1 Introduction

1.1 Techniques for analysing data from repeating surveys

This paper describes the use of composite estimation and state space modelling techniques for analysis of data from a repeating survey. The techniques take account of common sample between successive months and the resulting autocorrelation structure of the sampling error. The techniques are illustrated by an investigation of the effect of telephone interviewing in the Australian Labour Force Survey (LFS).

The problem of measuring the statistical impact of a methodology change in a repeated survey is a general one. The textbook approach to measuring the difference between two methodologies is to conduct a 'parallel run' - a concurrent survey using the new methodology. This is often too expensive to be practical. In the LFS situation the new methodology was gradually 'phased in' to the survey over seven months — this allowed estimation based on comparisons between portions of the survey using the different methodologies.

Approaches to measuring the statistical impact of a methodolgy change must account for the effects of the new methodology alongside other effects on the estimates. Typically the units surveyed at a particular time point have different histories in the survey. Some units are newly selected while others were selected at various previous time points. This leads to autocorrelations between specific subsets of the sample at different times. These autocorrelations are specifically accounted for in both the composite estimation and state space modelling techniques.

Section 2 describes the structure of the LFS and the data available to monitor the introduction of telephone interviewing. Section 3 describes an additive model for the error structure of the data that accounts for autocorrelations in the sampling error and for biases associated with how many times a group of dwellings has been surveyed. It describes estimation of these autocorrelations and biases based on data from before telephone interviewing was introduced. The section also introduces a number of models describing possible patterns of effect of telephone interviewing.

Section 4 describes an approach to designing composite estimators for additive effects, and presents composite estimators of the effect of telephone interviewing. Section 5 presents a state space model for a time series incorporating sampling error and bias effects. This approach was found to be very flexible and practical in the analysis of telephone interviewing. The results of the analysis are presented in section 6.

2 Measuring the impact of changes to the LFS

2.1 The Labour Force Survey sample

The LFS is a monthly household survey collecting information on the labour force status (employed, unemployed or not in labour force) of persons aged 15 or older. The sample contains over 30,000 dwellings and over 70,000 persons. The survey is designed to give very accurate estimates — for example, estimates of the employed as a proportion of the civilian population aged 15 or over have a standard error of under 0.2 percentage points.

The dwellings are selected by a multi-stage sampling scheme; geographic areas known as collectors' districts (CDs) are selected, then dwellings are selected within these CDs. The sample of CDs is divided into eight 'rotation groups' (RGs). Each month, the dwellings in one RG are rotated out of the sample, and replaced with dwellings from the same CDs. Thus selected dwellings remain in the survey for eight months and are then replaced by nearby dwellings.

2.2 Telephone interviewing

The LFS has until recently used face to face (FTF) interviewing, but from August 1996 telephone interviewing (TI) has been introduced.

Under the new method, FTF is used for the first time a set of dwellings is selected, and TI at later stages. In practice, not all dwellings agree or are able to be interviewed by telephone, so a rotation group using TI will actually use a combination of telephone and face to face interviews.

TI was introduced one RG at a time over the period August 1996 to February 1997. Starting in July 1996, every dwelling entering the survey was interviewed the first time by FTF, but invited to be interviewed by telephone in succeeding months. This approach led to a gradual phase-in of TI, with one RG using TI in August, two RGs using TI in September, and so on. By February, seven of the RGs were using TI, the remaining RG being the one with dwellings entering the survey for the first time. The period Augsut 1996 to February 1997 is called "the phase-in period".

2.3 Measuring the effect of telephone interviewing

Experience within the ABS and in other agencies suggests that changing the mode of interview has the potential to affect responses — see for example Kormendi and Noordhoek (1989) and Drew (1991). Published studies are not directly comparable to the Australian situation, and so there was little indication of what sort of effect could be expected.

The phased introduction of TI in the LFS allows the difference in estimates between TI and FTF methods to be measured. The difference will be referred to as "the TI effect". An early measure of the likely size of any TI effect was of great importance to users of the statistics.

The objective of this paper is to record methods used in analysing the effect of telephone interviewing on estimates of labour force status. The approaches used allow for considerable flexibility in modelling the TI effect and addressing issues such as whether the effect changed over time.

The analysis identified an effect on estimates from using the TI methodology over the early months of its introduction. The effect appears to have been transitory, with later months consistent with a zero effect.

2.4 Analysis at rotation group level

It was decided to evaluate the effect of TI by analysing estimates at RG level. Such estimates are sufficiently stable to allow standard methods of analysis of monthly data to be used. To analyse data at person level would be very complicated, particularly as even in a RG using TI the persons can elect to be interviewed FTF, and this choice may be related to labour force status.

Because the analysis is at RG level, the analysis measures not the effect of the actual interview mode, but the effect of the mix of telephone and FTF interviews that results from using TI in a RG. This leads straightforwardly to measures of the effect on published estimates.

2.5 Available estimates

Monthly estimates of persons by labour force status were obtained for each RG, categorised by month, sex, age (grouped as 15–19, 20–24, ... , 50–54, 55–64, 65+) and part-of-state (14 geographic regions covering Australia).

Within these categories, the estimates for each RG were pro-rated to match known population benchmarks. This "benchmarking" ensures that each RG represents a similar mix of individuals. This is important since the intention is to compare population estimates derived from RGs using TI with the estimates from other RGs using FTF.

Key estimates investigated were the proportion of the in scope population unemployed and the proportion employed, both by sex and overall. Data was available at RG level from January 1990 onwards.

2.6 Components of the analysis

Modelling the error structure

Knowledge of the structure of the time series up to July 1996 allows modelling of the situation before TI was introduced. This provides a baseline for evaluating whether TI had any effect.

A model describing the autocorrelation structure of rotation group estimates was fitted to the time series up to July 1996. All the estimates analysed were of rates or proportions rather than counts, since rates were assumed to have a reasonably constant variance and autocorrelation structure over the period from January 1990 on.

Designing a composite estimator

Simple estimators of the TI effect can be based on a comparison of the TI and non-TI RGs at a single time point. Composite estimators using data from a number of months were developed, achieving a much lower variance than the simple estimators. The composite estimators are based on an additive model and a model for the autocorrelation structure of the series.

Fitting time series models

The autocorrelation structure can be used to specify the sampling error component of a state space model for the time series. An additive model was used which incorporates a parameter for the TI effect.

This approach is very flexible, and was used to investigate various models for a TI effect that is not constant over time. Results from time series modelling were compared to those from composite estimation.

3 Modelling the error structure

The analysis depends on an understanding of the errors affecting the data, and especially the correlation structure of the sampling errors. The techniques for modelling and measuring these are described in this section.

3.1 An additive model for the series

This work uses an additive model that describes the series of estimates from each of the eight RGs. The series is decomposed into the true value of the item being measured, a bias associated with the survey procedures (in this case, TI or FTF interviewing), and a sampling error.

Each RG at a given time t is characterised by the number of times j the dwellings in the RG have been sampled. For example, in the RG with months-in-survey $j = 1$ the dwellings are being sampled for the first time.

Let y_t^j be the estimated value for the RG with months-in-survey j at time t. Let Y_t be the true value at time t. The following additive model at RG level incorporates sampling error e_t^j and two sources of bias, b^j (the months-in-survey effect) and T_t^j (the TI effect).

$$y_t^j = Y_t + b^j + T_t^j + e_t^j \qquad (1)$$

3.2 A model for the months-in-survey effect

Even before introducing TI there was an established months-in-survey effect on RG estimates. This effect captures the tendency for RGs containing dwellings sampled for more times to report somewhat lower employment and unemployment (and higher numbers not in the labour force). A simple model assumes that the bias for months-in-survey j is constant over time at a value b^j known as the months-in-survey effect, and that the net effect of these biases across all values of j was 0. A simple estimate based on the mean over N time points before TI was introduced would be

$$\hat{b}^j = \tfrac{1}{N}\Sigma_{t=1}^{N}(y_t^j - y_t) \quad \text{for } y_t = \tfrac{1}{8}\Sigma_{j=1}^{8} y_t^j \ . \qquad (2)$$

Table 1 shows estimates of this months-in-survey effect based on data from January 1990 to July 1996. It appears that estimates of both employment and unemployment tend to be lower for RGs with dwellings surveyed for more times.

Table 1: Effect of months-in-survey on rotation group estimates
Simple estimate (percentage points)

LFS estimate	Months-in-survey of rotation group							
	1	2	3	4	5	6	7	8
Proportion employed	0.15	0.08	0.05	0.01	–0.06	–0.09	–0.04	–0.09
Proportion unemployed	0.14	0.05	0.00	–0.01	0.00	–0.02	–0.07	–0.08

These estimates were used to adjust for the months-in-survey effect in the composite estimation approach, as described in Section 4. In the state space modelling approach, values for the b^j parameters were estimated simultaneously with other parameters in the model — see Section 5. In using the state space modelling approach the months-in-survey biases can be allowed to vary through time. Given the limited span of data available this was not pursued, since the months-in-survey biases must be assumed to change slowly or not at all if changing biases over the phase-in period are to be treated as the effects of using TI.

3.3 Models for the TI effect

The effect of the TI method on RG estimates is modelled as additional to the established months-in-survey bias. Denote by T_t^j the change in bias that results from the TI effect for RG j at time t under TI. T_t^j is zero for RGs not using TI, and possibly non-zero for RGs using TI. Four different models for the TI effect for RGs using TI are presented here.

Constant effect model M$_0$

In the first few months of use of the TI approach the main concern was to identify if there was any evidence of a significant TI effect. To this end a model was proposed in which every RG using TI was affected by the same additive bias T. In this model $T_t^j = T$.

Monthly effect model M$_1$

It is also important to obtain some picture of whether the TI effect is changing with time. A simple approach to this question is to use a model with separate TI effects for each month. These can then be inspected to identify any patterns over the months. In this monthly effect model, $T_t^j = T_t$ a separate constant for each time t.

Two-level effect model M$_2$

From inspection of estimates from the monthly effect model, it became apparent that earlier months up to November 1996 indicated a TI effect but that in later months the effect appeared reduced. To address the issue of whether there was a significant change after November a two parameter model was used, in which

$$T_t^j = T_{\text{INITIAL}} \quad \text{for } t \text{ up to November 1996}$$
$$= T_{\text{FINAL}} \quad \text{for } t \text{ from December 1996 onward.} \qquad (3)$$

In this model a significant value of T_{FINAL} would indicate that the long-term TI effect was significantly different from zero. A significant value of

$T_{\text{DIFF}} = T_{\text{INITIAL}} - T_{\text{FINAL}}$ would indicate that the TI effect had changed significantly from the early months to the later months.

Transitory effect model M_3

One further model was introduced which assumes a constant TI effect up to November 1996 which then decreased gradually to 0 in February 1997. This model can be written $T_t^j = c_t T_{\text{TRANS}}$ for c_t taking the value 1 up to November 1996 and decreasing gradually to 0 by February 1997.

3.4 Modelling autocorrelated sampling error

This section outlines the notation and calculations for measuring standard error and autocorrelations at RG level. It is a summary of methods presented in Bell and Carolan (1998).

Basic structure of the sampling error

A model is used to describe the sampling error structure of the rotation groups. The model assumes that sampling errors from different rotation groups have a common standard error and are uncorrelated. For the same RG, the correlation between sampling errors from different survey months t_1 and t_2 depends only on the gap between the times $|t_2 - t_1|$ and the number of occasions the dwellings in the RG have been selected (which determines if the two survey months have the same sample of dwellings).

The sampling error is assumed to have $\text{E}(e_t^j) = 0, \text{Var}(e_t^j) = 8\sigma_E^2$ and autocorrelations for lag $k > 0$, $1 \le j \le 8$ and $1 \le m \le 8$ given by

$$\begin{aligned} \text{Corr}(e_t^j, e_{t-k}^m) &= \rho_k^j \quad \text{if } m = j - k + 8i \text{ for integer } i > 0 \text{ (ie. same RG)} \\ &= 0 \quad \text{otherwise (ie. different RGs) .} \end{aligned} \tag{4}$$

Estimating variance and autocorrelations

The variances and the sampling error autocorrelations were estimated using N=79 months of RG estimates from January 1990 to July 1996. Separate estimates are obtained for each time in survey j and lag m, using the following formulae, based on Bell and Carolan (1998).

Define the pseudo-error for the RG with dwellings sampled for the jth time to be $\hat{e}_t^j = y_t^j - \hat{b}^j - y_t$. The unbiased estimate of variance for y_t assuming variance is constant over time is then given by

$$\hat{\sigma}_E^2 = \tfrac{1}{N} \Sigma_{t=1}^{N} \tfrac{1}{56} \left(\sum_{j=1}^{8} \hat{e}_t^j \right) \; . \tag{5}$$

Estimates of the autocovariances of the pseudo-errors at lag k are given by

$$\hat{C}_k^j = \tfrac{1}{N} \sum_{t=k+1}^{N} \left(\hat{e}_t^j, \hat{e}_{t-k}^{j-k+8i} \right), \; k = 0, 1, 2, \ldots \tag{6}$$

for integer i chosen so that $1 \le j - k + 8i \le 8$.

Estimates of the pseudo-error autocorrelations ρ^j_{Pk} at rotation group level are given by

$$\hat{\rho}^j_{Pk} = \hat{C}^j_k / \hat{C}^j_0 \,. \tag{7}$$

It is straightforward to show that these estimates are biased as estimates for the true autocorrelations. In fact, their expected values are given by $E(\hat{\rho}^1_{Pk}, \ldots, \hat{\rho}^8_{Pk}) = (\rho^1_k, \ldots, \rho^8_k)A$, for A an 8 by 8 matrix with diagonal elements $\frac{49}{56}$ and off-diagonal elements $\frac{1}{56}$. This leads to using corrected estimates $\hat{\rho}^j_k$ of the autocorrelation at lag k for estimates from the RG in survey for the jth time, given by

$$(\hat{\rho}^1_k, \ldots, \hat{\rho}^8_k) = (\hat{\rho}^1_{Pk}, \ldots, \hat{\rho}^8_{Pk})A^{-1} \,. \tag{8}$$

3.5 A model to smooth the autocorrelations

The autocorrelation estimates are highly variable; this variability can be reduced by fitting to the estimates a model with a small number of parameters. Assume that the sampling error autocorrelation in a rotation group depends only on the lag and on whether the RG has a common sample of dwellings between the two time points. This leads to the model

$$\begin{aligned} \rho^j_k &= \rho_{Wk} \quad \text{if } j > k \ \text{(ie. same dwellings)} \\ &= \rho_{Bk} \quad \text{otherwise (ie. different dwellings)} \,. \end{aligned} \tag{9}$$

It would be possible to obtain estimates of ρ_{Wk} as an average of the estimates $\hat{\rho}^j_k$ for which $j > k$, and of ρ_{Bk} as an average of the other lag k autocorrelations. An alternative is to smooth the autocorrelations using a model, so as to improve their stability and to enforce simple relationships, such as autocorrelations decreasing with increases in the lag.

A four parameter model was introduced for the autocorrelations. The parameters r_U, r_P, θ_P and θ_B have an interpretation in the context of the state space modelling that will be presented in section 5. Under this model the autocorrelations are given by

$$\rho_{Wk} = (1 - r_U^2)(\theta_P{}^k r_P^2 + \theta_B{}^k(1 - r_P^2)) \text{ and} \tag{10}$$

$$\rho_{Bk} = (1 - r_U^2)\theta_B{}^k(1 - r_P^2) \,. \tag{11}$$

A simpler three parameter model (with $r_U = 0$) was rejected because it did not fit the estimated autocorrelations well.

The parameters were fitted so that for lags 1 to 8 the autocorrelations from the model were as close as possible to the estimated autocorrelations. Closeness was defined by least squares distance. The parameters were chosen to minimise the distance function

$$\Sigma^8_{j=1}(\Sigma^{j-1}_{k=1}(\hat{\rho}^j_k - \rho_{Wk})^2 + \Sigma^8_{k=j}(\hat{\rho}^j_k - \rho_{Bk})^2) \,. \tag{12}$$

The optimal parameters were chosen by a numerical search procedure, restricting r_U, r_P, and θ_P to be in [0, 1] and restricting θ_B to [0.94, 0.98]. (This restriction ensures that any long term autocorrelation ρ_{Bk} decreases slowly with increasing lag. Some restriction is necessary to ensure a single best solution in situations where ρ_{Bk} is near zero.)

The resulting smoothed autocorrelation estimates are presented in table 2 along with estimated standard errors.

Table 2: Estimated standard errors and rotation group autocorrelations

Lag (months)	Proportion unemployed σ_E =0.11 (% points)		Proportion employed σ_E =0.21(% points)	
k	ρ_{Wk}	ρ_{Bk}	ρ_{Wk}	ρ_{Bk}
1	0.62	0.11	0.80	0.15
2	0.52	0.11	0.71	0.15
3	0.44	0.10	0.64	0.14
4	0.37	0.09	0.57	0.13
5	0.31	0.09	0.50	0.12
6	0.26	0.08	0.45	0.11
7	0.22	0.08	0.40	0.11
8	n.a.	0.07	n.a.	0.11

Standard error is σ_E
Autocorrelations between estimates from the same RG at lag k are ρ_{Wk} within a set of dwellings and ρ_{Bk} between different sets of dwellings.
n.a. not applicable

4 Composite estimation

4.1 Simple estimates of a constant TI effect

Discounting the months-in-survey effect

Estimates of the TI effect were based on comparing estimates from different RGs. Since RG estimates are affected by a months-in-survey effect this needs to be removed to avoid affecting the estimates of the TI effect. To this end, the formulae in this section will be based on RG estimates $\tilde{y}^j_t = y^j_t - \hat{b}^j$ that subtract out the estimate of the months-in-survey effect.

Level-based estimate

Suppose the TI effect depends only on the month, so that $T^j_t = T_t$. An estimate of this TI effect based on comparing TI and non-TI groups at a single time point will be referred to as the level-based estimate. Let G(t) be the set containing the months-in-survey values for RGs using TI at time t, and let #G(t) be the number of elements of this set. Then the level-based estimate is given by

$$\hat{T}_{\text{LEV},t} = \frac{1}{\#\text{G}(t)} \Sigma_{j\in \text{G}(t)} \tilde{y}^j_t - \frac{1}{8-\#\text{G}(t)} \Sigma_{j\notin \text{G}(t)} \tilde{y}^j_t \,. \quad (13)$$

Movement-based estimate

Another estimate can be based on the difference (or movement) between successive monthly estimates from a given RG. For RGs having the same sample of dwellings at lag 1 there is a high autocorrelation between successive estimates, so the movement can have a lower variance than the RG estimates themselves.

To obtain an estimate of TI effect based on the movement, note that in any month after July 1996 one of the RGs will have been interviewed using FTF last month and using TI that month. The expected value of the movement in this RG is thus the true movement $Y_t - Y_{t-1}$ plus the TI effect for that month. So an estimate of the TI effect can be obtained by subtracting from this RG's movement an estimate of the true movement.

The true movement is estimated by the movement in the other RGs that have common dwellings at lag 1. Two versions arise. The first 'monthly' version estimates the true movement using the non-TI RGs only. The second 'constant' version uses both TI and non-TI RGs — this gives a lower standard error, but is only appropriate under that assumption that the TI effect is constant. These two movement-based estimators are given by

$$\hat{T}_{\text{Monthly},t} = (\tilde{y}_t^2 - \tilde{y}_{t-1}^1) - \frac{1}{7-\#G(t)} \Sigma_{j \notin G(t), j \geq 3} (\tilde{y}_t^j - \tilde{y}_{t-1}^{j-1}) \text{ and} \tag{14}$$

$$\hat{T}_{\text{Constant},t} = (\tilde{y}_t^2 - \tilde{y}_{t-1}^1) - \tfrac{1}{6} \Sigma_{j \geq 3} (\tilde{y}_t^j - \tilde{y}_{t-1}^{j-1}) . \tag{15}$$

Table 2 gives level-based and movement-based estimates of the TI effect for each month of the phase-in period. Estimates are marked with asterisks to indicate whether they are significantly different from zero.

Table 3: Simple estimates of TI effect based on a single months data

Level based and lag 1 movement based estimates of TI effect

Month	Proportion unemployed			Proportion employed		
	Level-based	Movement-based "Monthly"	Movement-based "Constant"	Level-based	Movement-based "Monthly"	Movement-based "Constant"
Aug 1996	0.01	0.09	0.09	0.27	–0.73*	–0.73*
Sep 1996	0.53*	0.71*	0.60*	–0.93	–0.04	–0.03
Oct 1996	0.26	–0.12	0.00	–0.61	–0.81*	–0.79*
Nov 1996	–0.01	0.09	0.13	–0.61	–0.25	–0.27
Dec 1996	0.01	0.14	0.04	–0.30	–0.60	–0.82*
Jan 1997	–0.31	–0.08	0.03	0.37	0.05	–0.22
Feb 1997	–0.12	n.a.	–0.27	0.89	n.a.	0.04

* significant at p<0.1 level, ** significant at p<0.02 level,
n.a. not applicable

The standard errors on these simple estimates are large. The only obvious feature is that the TI effect estimates for proportion employed tend to be negative. To test this requires combining information across months.

Combining the estimates

Suppose the TI effect is constant (ie. $T_t = T$). Then a suitable weighted average of the simple estimators of T given above will have a lower variance than the individual estimates. The appropriate weighted average to use is that which minimises the variance. Other simple estimates based on differences between estimates at lag two (or more) can also be devised, and these can be included in the weighted average.

The variance of each simple estimator and of any proposed weighted average can be evaluated under the model since the variance-covariance structure of the RG estimates is known. The calculations that implement this idea are an application of the more general formulation discussed below.

4.2 Designing a composite estimator

Variance of a linear combination of RG estimates

Assume that data from $L+1$ months is to be used for the composite estimator. Write $\tilde{y}$ and α as the column vectors with elements $\{\tilde{y}^j_{t-l}\}$ and $\{\alpha^j_{t-l}\}$ respectively for $j = 1,\ldots,8$ and $l = 0,\ldots,L$, where the elements of α lie in the interval [-1,1]. The general formula for the variance of a linear combination of the RG estimates is given by

$$\text{var}(\alpha'\tilde{y}) = \alpha'\text{var}(\tilde{y})\alpha \,. \qquad (16)$$

Here var($\tilde{y}$), the covariance matrix of the RG estimates, is known from the variance and covariances given in table 2. This matrix calculation can be used to derive the variance of the simple estimators described in Section 4.1.

A composite estimator of constant TI effect

The expected value of a linear combination of the $\tilde{y}^j_t$ under the model (1) is

$$E(\alpha'\tilde{y}) = \Sigma^L_{l=0}\{Y_{t-l}(\Sigma^8_{j=1}\alpha^j_{t-l}) + \Sigma_{j\in G(t-l)}T^j_{t-l}\alpha^j_{t-l}\}\,. \qquad (17)$$

(for G defined as in 4.1) since $\Sigma^8_{j=1} b^j = 0$ (by definition) and $E(e^j_t) = 0$.

For the linear combination $\alpha'\tilde{y}$ to be an unbiased estimator of T the following constraints must be placed on the choice of α.

$$\Sigma^8_{j=1}\alpha^j_{t-l} = 0 \qquad \text{for } l = 0,\ldots,L, \text{ and} \qquad (18)$$

$$\Sigma^L_{l=0}\Sigma_{j\in G(t-l)}\alpha^j_{t-l} = 1\,. \qquad (19)$$

Under these constraints, $E(\alpha'\tilde{y}) = T$ under the model with constant TI effect. The optimal composite estimator based on this data will be the choice of α that minimises the variance (16) under these constraints.

The optimum choice of α was obtained using standard results for minimisation of a quadratic form under linear constraints (see for example Rao (1973) p. 65). Writing the constraints in the form $C'\alpha = c$ and setting V=var($\alpha'y$), the optimal α is given by $\alpha^* = V^{-1}CQ^{-}c$, for Q^{-} any generalised inverse of $(C'V^{-1}C)$.

Composite estimators of parameters in other TI models

A similar approach was applied to the other models for the TI effect to give composite estimators for the various parameters in these models. The process was to substitute the appropriate TI effect model into the expectation formula (17) and then to minimise variance (16) under constraints that ensure that the expectation equals the parameter being estimated.

For example, for the two level model M_2 under the constraints (18) the expectation of an estimator $\alpha'\tilde{y}$ for t corresponding to February 97 and L=7 is given by

$$E(\alpha'\tilde{y}) = T_{\text{INITIAL}}\Sigma^6_{l=3}(\Sigma_{j\in G(t-l)}\alpha^j_{t-l}) \;+\; T_{\text{FINAL}}\Sigma^2_{l=0}(\Sigma_{j\in G(t-l)}\alpha^j_{t-l})\,. \qquad (20)$$

So a minimum variance estimate of T_{FINAL} is obtained by minimising variance (16) subject to the constraints (18) and the additional constraints $\Sigma_{l=3}^{6}(\Sigma_{j\in G(t-l)}\,\alpha_{t-l}^{j}) = 0$ and $\Sigma_{l=0}^{2}(\Sigma_{j\in G(t-l)}\,\alpha_{t-l}^{j}) = 1$. The resulting composite estimators under the various models of TI effect are presented in table 4.

Table 4: Composite estimators under different models of TI effect, proportion unemployed, and proportion employed by sex

Model of TI effect	Proportion unemployed	Proportion employed Male	Female	Total
M_0 Constant to Feb 1997	0.12	–0.66**	–0.07	–0.38**
M_1 Monthly: Aug 1996	0.01	–1.07**	0.07	–0.51
Sep 1996	0.56*	–0.71*	–0.25	–0.46*
Oct 1996	0.21	–1.34**	0.00	–0.64**
Nov 1996	0.06	–0.83**	–0.55*	–0.69**
Dec 1996	0.11	–0.23	–0.33	–0.30
Jan 1997	–0.16	–0.47	0.60	0.06
Feb 1997	–0.07	1.01	0.42	0.71
M_2 Two level: T_{FINAL}	0.01	–0.19	0.13	–0.06
Two level: T_{DIFF}	0.18	–0.78**	–0.32	–0.51*
Two level: T_{INITIAL}	0.19*	–0.97**	–0.19	–0.57*
M_3 Transitory effect	0.21*	–0.97**	–0.26	–0.60**

* significant at p<0.1 level, ** significant at p<0.02 level

The effects on proportion employed for males are clear, with a consequent effect on the total proportion employed.

4.3 Nonparametric confidence intervals

Each of the estimators described above is simply a linear combination of the RG estimates at one or more successive time points. Applying the estimator to every such set of time points before July 1996 gives a number of (autocorrelated) values, each of which has the same distribution as the final composite estimator, under the model and assuming no TI effect.

The empirical distribution of these values can therefore be used to estimate the distribution of the estimator assuming there is no TI effect. This leads to a non-parametric confidence interval for the estimate of the TI effect and a non-parametric significance test of whether TI effect is 0.

The non-parametric confidence intervals for the statistics have a high variability, as not very many observed values are available (one for each time point), and these observations are quite highly correlated over time. However, on average over the various parameters estimated the confidence intervals appear to be only slightly larger than the (parametric) confidence intervals predicted from the estimated variance and autocorrelations. The parametric estimates of standard error appear to be 1 or 2 per cent too small, and this should be kept in mind when considering tests of significance that give borderline results.

5 Fitting a time series model

5.1 State space models

In a state space model, the observed data at a time point depends on a vector of unobserved true states — the relationship is given by the *observation equation*. The unobserved states are modelled as a Markov Chain, with states at one time depending on those at the previous time, as given by the *state equation*. The dependencies between the state vectors are used to model the autocorrelation between the observed values at successive time points.

A linear Gaussian state space model has the observed data and the new state vector as a linear function of the previous state vector plus a linear function of a vector of i.i.d. N(0, 1) variables known as *innovations*. For such a state space model the least squares estimates of the states at each time point are obtained by using the smoothed Kalman filter. In this study the diffuse Kalman filter of de Jong (1990) was used.

Advantages of state space modelling

A state space model is a very flexible model for investigating time series data. It is a natural model to use when there is interest in the unobserved components of a time series, or when some parameters may change over time. A change of model at any time point can be easily handled — for instance, to deal with telephone interviewing effects of varying kinds. It is also possible to deal automatically with outliers in the model by using innovations that are a mixture of Gaussians - see Bell and Carolan (1998) for details.

5.2 A state space model for sampling error

This paper uses the state space model for sampling error introduced in Bell and Carolan (1998). The model decomposes the sampling error e_t^j into three parts:

$$e_t^j = B_t^j + P_t^j + U_t^j \ . \qquad (21)$$

B_t^j is a component which has a very high autocorrelation θ_B with the same component from the same RG in the previous month. This component is responsible for the long term autocorrelation within a RG.

P_t^j is a component which has high autocorrelation θ_P with the same component from the same RG in the previous month, except when the dwellings surveyed have changed between the two months. This component is responsible for the autocorrelation due to a common sample of dwellings between months.

U_t^j is a component of the sampling error with no autocorrelation.

The proportion of the variance of the sampling errors that is explained by the component U_t^j is given by r_U^2; the proportion explained by component P_t^j is given by $(1-r_U^2)r_P^2$; and the remaining proportion explained by B_t^j is given by $(1-r_U^2)(1-r_P^2)$. Thus the model depends on five parameters,σ_E^2, r_U, r_P, θ_P and θ_B, the values of which are estimated by methods given in section 3.5. The state equations for the model are as follows:

$$B_t^j = \begin{cases} \left(8\sigma_E^2\left(1-\theta_B^2\right)\left(1-r_P^2\right)\left(1-r_U^2\right)\right)^{1/2} u_{Bt}^j & \text{for } t=1 \\ \theta_B B_{t-1}^8 + \left(8\sigma_E^2\left(1-\theta_B^2\right)\left(1-r_P^2\right)\left(1-r_U^2\right)\right)^{1/2} u_{Bt}^j & \text{for } j=1,\ t>1 \\ \theta_B B_{t-1}^{j-1} + \left(8\sigma_E^2\left(1-\theta_B^2\right)\left(1-r_P^2\right)\left(1-r_U^2\right)\right)^{1/2} u_{Bt}^j & \text{for } j=2,\dots,8,\ t>1 \end{cases}$$

$$P_t^j = \begin{cases} \left(8\sigma_E^2 r_P^2\left(1-r_U^2\right)\right)^{1/2} u_{Pt}^j & \text{for } j=1 \text{ or } t=1 \\ \theta_P P_{t-1}^{j-1} + \left(8\sigma_E^2\left(1-\theta_P^2\right) r_P^2\left(1-r_U^2\right)\right)^{1/2} u_{Pt}^j & \text{for } j=2,\dots,8,\ t>1 \end{cases}$$

$$U_t^j = \left(8\sigma_E^2 r_U^2\right)^{1/2} u_{Ut}^j \qquad \text{for } j=1,\dots,8 \qquad (22)$$

The innovations in the model, $u_{Bt}^j, u_{Pt}^j, u_{Ut}^j$ are assumed to be i.i.d $N(0,1)$. The multipliers applied to the innovations ensure that $\mathrm{var}(e_t^j) = 8\sigma_E^2$.

A new sample of geographic areas was introduced over the last four months of 1992. This leads to zero autocorrelation for particular combinations of j and t. In the state space modelling approach this is accounted for by using the equations applicable to $t=1$ at the time when the new sample was introduced to a RG.

5.3 The overall state space model

Modelling the true values

To complete the state space model requires a model for the dependencies in the true series Y_t. It is usual to model such dependencies using a decomposition into trend, seasonal and irregular, or as an ARIMA model. This was not done in this study, except as a subsidiary experiment (see section 6.6). Instead, a simpler approach (similar to that of composite estimation) was used in which the true values are effectively unrelated from time to time. The state equation associated with this is given by

$$Y_t = Y_{t-1} + \tau_Y u_{Yt} \,. \qquad (23)$$

for $u_{Yt} \sim$i.i.d $N(0,1)$ and τ_Y large (eg. $\tau_Y = 100$ for data expressed in percentage points). This model does not borrow any strength from the likely relationship between true values across time, and so may result in estimates of the TI effect with somewhat higher standard errors than would have been obtained by assuming a more sophisticated model for the true values.

The state space model

The overall state space model is given by the equations (1), (21), (22) and (23), and can be written out as

$$y_t^j = Y_t + b^j + T_t^j + B_t^j + P_t^j + U_t^j \,. \qquad (24)$$

The state vector at time t contains Y_t with state equation (23) and B_t^j and P_t^j with state equations (22). The U_t^j appear only in the observation equation and do not require state equations.

The months-in-survey effects b^j are treated as regression parameters for $j=1,\dots 7$ and the restriction $b^8 = -\Sigma_{j=1}^7 b^j$ is imposed to force the month in survey effects to add to 0. This is straightforward using the diffuse Kalman filter. The parameters in the TI effect model could be treated as regression parameters, but were actually implemented by introducing an extra state for each parameter. For the constant TI effect model this simply means including the state T_t in the state vector and adding the state equation $T_t = T_{t-1}$ to the set of state equations.

Initialising the states

To complete the definition of the model, ready for estimation using the Kalman filter, initial distributions (i.e. before observing any data) for the regression parameters and for the states at time t=1 must be specified. These distributions are known as priors. The regression parameters were given diffuse priors, corresponding to total uncertainty about the intial value. This situation is handled within the diffuse Kalman filter (using the limit as the variance of the prior tends to infinity). The true value and TI effect states could be given diffuse priors, but it was computationally cheaper to give the initial values for these a normal distribution with large variance.

The components of the sampling error B_t^j and P_t^j have a zero expectation and a known variance as given in (22) for time $t=1$. This gives a prior distribution to be used for these states. A diffuse prior for these components is not appropriate and could give unrealistic results.

5.4 State space model estimates of constant TI effect

Table 5 presents estimates of constant TI effect based on the state space modelling approach.

Table 5: State space model estimators under different models of TI effect, proportion unemployed, and proportion employed by sex

Model of TI effect	Proportion unemployed	Proportion employed: Male	Female	Total
M_0 Constant to February	0.09	−0.61**	−0.05	−0.35*
M_1 Monthly: Aug 1996	−0.01	−1.04**	0.07	−0.49
Sep 1996	0.54**	−0.66*	−0.24	−0.44
Oct 1996	0.19	−1.28**	0.02	−0.61**
Nov 1996	0.03	−0.80**	−0.53*	−0.67**
Dec 1996	0.09	−0.19	−0.30	−0.27
Jan 1997	−0.17	−0.44	0.61	0.07
Feb 1997	−0.12	1.12*	0.48	0.78
M_2 Two level: T_{FINAL}	−0.01	−0.14	0.15	−0.04
Two level: T_{DIFF}	0.18	−0.78**	−0.33	−0.50*
Two level: $T_{INITIAL}$	0.17	−0.92**	−0.18	−0.54*
M_3 Transitory effect	0.19	−0.93**	−0.25	−0.58**

* significant at p<0.1 level, ** significant at p<0.02 level

Note that the estimates are very similar to those for composite estimation. The sampling errors are comparable also, tending to be a little lower. This is a useful check on the validity of the analyses. The results of the analyses are discussed in the next section.

6 The telephone interviewing investigation

6.1 Investigation of impact of TI on estimates

The results in this section follow the investigation of the TI effect over the period of the phase-in of telephone interviewing. An investigation was conducted every month from August 1996 to February 1997, with methods being refined each month as results opened up new questions. The key method used was the state space modelling approach, and numbers presented in this section are from that approach. Composite estimates and the simple level- based and movement-based estimates were also examined each month to check the results. Plots showing the rotation group estimates by time were also important in explaining the cause of particular effects and in looking for patterns.

The results below are given using all the data up to February 1997. This makes for a straightforward presentation, but hides somewhat the full variety of investigations conducted over the phase-in period. Each month the investigations, including new models prompted by the new data, were completed by the day after the data became available. This underlines the easy application of the modelling techniques used once the initial investment in implementing the methods has been made.

6.2 Detecting a TI effect

Early in the phase-in period there was little data available from TI RGs, and the focus of investigation was on establishing if the TI method affected any of the series. The focus was therefore on the constant TI effect model M_0.

By the time November 1996 data was analysed it appeared that the TI RGs were likely to report somewhat lower employment than the FTF RGs. In the December 1996 issue of the *Labour Force Australia* publication, the ABS issued information about the apparent effect of TI on employment estimates, and the range of values quoted was based on the model M_0.

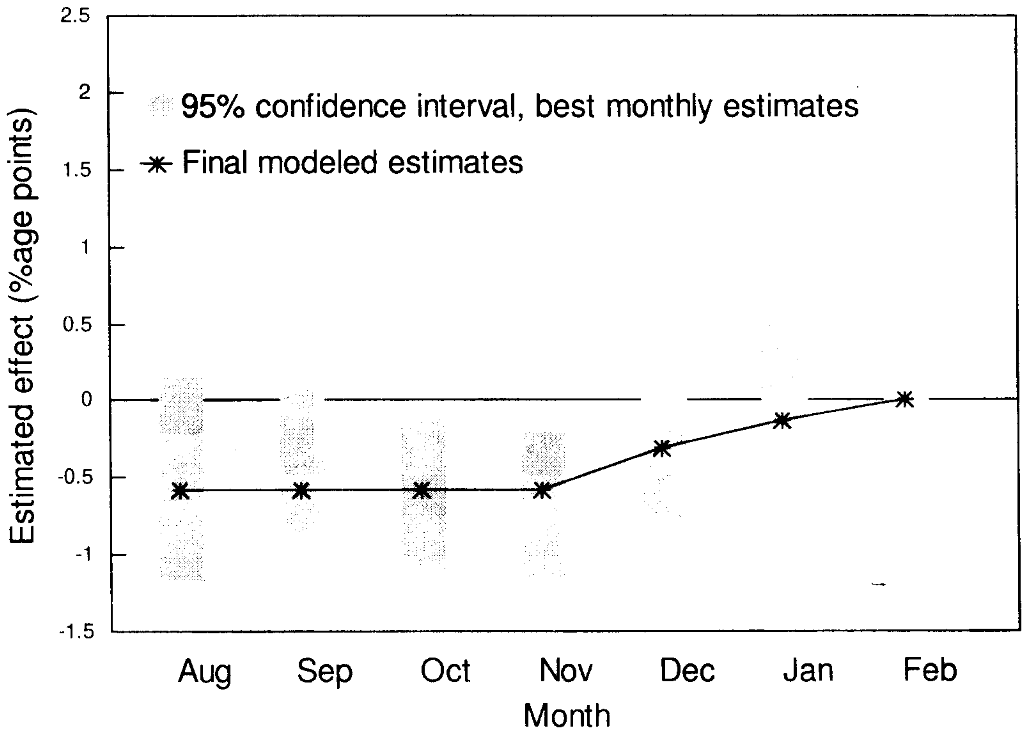
6.3 Behaviour of the TI effect over time

As more months of data were accumulated, attention turned to whether the TI effect was changing over time. The discussion below will focus on the TI effect on proportion employed.

Graph 1 displays a grey bar for each month representing a 95% confidence interval for the TI effect on a RG estimate of proportion employed. These are based on model M_1, which allows the TI effect to vary from month to month, using the state space modelling approach. The true TI effect for a month is expected to lie in the grey region with 95% probability.

The grey bars tend to be below zero for the period from August 1996 to November 1996, but from December 1996 they do include zero. This, and similar graphs for males only, suggests that the TI effect may have declined since November 1996.

Graph 1: Estimates of effect of telephone interviewing on an RG under TI, proportion employed, by month



'Best monthly estimates' are based on state space modelling for model M_1, 'Final modelled estimates' are based on state space modelling for model M_3

6.4 Has the TI effect changed from December 1996 onwards?

A model was fitted on data up to February 1997 that proposed a TI effect that was constant from August to November 1996 and then constant at a different level for December 1996 onwards. Results for this model M_2 confirmed that the August to November TI effect on employed persons was highly significant. Importantly, it also established that the TI effect from December onwards was significantly different (at the $p<0.02$ significance level) from the August to November value. The December-onwards level of the TI effect was small and not significantly different from zero.

6.5 Model based on a transitory effect

There was some concern that the assumption that the TI effect dropped to zero within a single month (i.e. between November and December 1996) was unrealistic. The transitory effect model M_3 was proposed in which the TI effect would be constant from August to November and then decrease gradually to zero by February, in such a way that the estimated total impact of TI on estimates would decrease linearly. This model fitted the data better than the model M_2 which proposed an abrupt change occurring after November.

It is considered that this model provides the best summary available of the effect of TI over the phase-in period. The estimates of the TI effect for each month from this final model are shown in graph 1. These effects apply to each RG under TI in the month given — because the number of RGs under TI increased through the period, they translate into an overall effect that increased to November and then decreased thereafter.

6.6 Other investigations

The conclusion from these investigations was that the data is best explained by a transitory TI effect that had subsided to near zero by February 1997. In arriving at this position, other possibilities were also examined.

Does the TI effect change with months-in-survey?

One model investigated was that telephone interviewing did not change over time, but instead affected different RGs according to the number of times the dwellings had been surveyed. On the basis of data up to December 1996 this model fitted fairly well, with some evidence that the effect was greater for the first few times a RG was surveyed. The model did not fit as well as the monthly effect model M_1.

Using all the data up to February 1997 it was clear that this model was inferior to the model of a transitory TI effect. It is possible that there remains some effect of time in survey, but it is too small to be estimated reliably. Many more months of data will be required to obtain a good picture of what time in survey effects apply under the TI methodology.

Robustness of the results to outliers

Another issue was whether outliers in the RG data were producing the appearance of a TI effect. This was addressed in three simple ways. The first was to exclude the most influential RG and month estimate, by giving it a separate parameter. The second way was to assume that a specific RG had an additional impact on the estimates for a series of months. In both these experiments the estimated TI effect decreased, but was still quite significant.

The third way was to examine whether any of the innovations over the period of TI introduction were large (defined as over 2.5 for this model where the innovations are distributed N(0, 1)). Such values would be highly influential, and would be better treated as outliers or using non-Gaussian state space modelling as montioned in Section 5.1. This examination did not identify any outliers during the TI phase-in period.

Decomposing the true value into trend, seasonal and irregular

The study used a simple model for the true value that gave no penalty to large movements between successive months. It was mentioned previously that an alternative approach would have been to impose a model for the true value that accounts for its trend and seasonality.

To look at the effects of this, the model for the true value Y_t was replaced by a state space decomposition into trend, seasonal and irregular as given in Bell and Carolan (1998). The model used is known as the Basic Structural Model (BSM) with local linear trend, to distinguish it from the 'free' model described by equation (23). In the BSM model, the state equation (23) for the true value is replaced by the decomposition

$$Y_t = L_t + S_t + I_t \tag{25}$$

and state equations for the three components are introduced:

$$\begin{aligned} L_t &= 2L_{t-1} - L_{t-2} + \tau_L u_{Lt} && \text{Trend component,} \\ S_t &= -\textstyle\sum_{j=1}^{11} S_{t-j} + \tau_S u_{St} && \text{Seasonal component,} \\ I_t &= \tau_I u_{It} && \text{Irregular component.} \end{aligned} \tag{26}$$

for innovations u_{Lt}, u_{St} and u_{It} i.i.d. N(0,1). This approach was applied to the analysis of proportion employed for each sex and in total. The variance parameters in the decomposition (defining for example how smooth the trend is) were set to values ($\tau_L = 0.018, \tau_S = 0.001$ and $\tau_I = 0.18$) suitable for a decomposition of the Australian series.

Results for the transitory effect model M_3 based on the free and BSM models of the true value are given in table 6. This experiment resulted in the BSM model giving estimates of TI effect with standard errors about 10 per cent lower than the free model. The cost of these improved SEs is that one has to make more assumptions about the structure of the true value.

Table 6: Estimate and standard error of transitory TI effect T_{TRANS}, proportion employed, by model for true value, by sex

	Free model for Y_t		BSM model for Y_t	
	Estimate	standard error	Estimate	Standard error
males	–0.93**	0.22	–0.84**	0.20
females	–0.25	0.23	–0.34	0.21
total	–0.58**	0.17	–0.58**	0.16

Using the BSM model would have given slightly different estimates of the TI effect, but the conclusions of the investigation would have been unchanged, with both models showing significant TI effects.

7 Conclusion

The main conclusion of these investigations is that in repeated surveys it can be worthwhile to use models that explicitly account for autocorrelations in the survey estimates. This paper has described methods to do this in the context of the investigation of telephone interviewing in the Australian Labour Force Survey.

These methods can be applied to the analysis of repeating surveys where overlap is controlled and where separate estimates can be generated for groups of sampled units that have different rotation histories. In the case presented here this was achieved by classifying estimates by LFS rotation group.

The investigation of telephone interviewing identified a transitory effect on estimates by Labour Force status during the period of the phase in. This effect appears to have diminished to near zero by the end of the phase-in period.

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